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ABSTRACTS OF TALKS GIVEN AT THE 4TH INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS*

(*Translated by A. R. Alimov*)

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The Fourth International Conference on Stochastic Methods (ICSM-4) was held June 2–9, 2019 at Divnomorskoe (near the town of Gelendzhik) at the Raduga sports and fitness center of the Don State Technical University, where the previous conference (ICSM-3) took place in 2018. ICSM-4 was organized by the Steklov Mathematical Institute of Russian Academy of Sciences (Department of Theory of Probability and Mathematical Statistics); Moscow State University (Department of Probability Theory); National Committee of the Bernoulli Society of Mathematical Statistics, Probability Theory, Combinatorics, and Applications; Peoples' Friendship University of Russia; and the Don State Technical University (Department of Higher Mathematics). The conference chairman was A. N. Shiryaev, a member of the Russian Academy of Sciences, who chaired the previous three conferences and also headed the Organizing Committee and the Program Committee.

Many leading scientists from Russia, France, Germany, Portugal, and Bulgaria took part in ICSM-4. Russian participants came from Voronezh, Zernograd, Kaluga, Maikop, Nizhni Novgorod, Rostov-on-Don, Samara, St. Petersburg, Taganrog, Ufa, and Khabarovsk. Approximately one-quarter of the talks were given by postgraduate and undergraduate students. Twenty-one talks were given at joint sessions, and 44 talks were presented at parallel sessions.

The Local Organizing Committee headed by I. V. Pavlov successfully managed the logistics of the conference. The participants recognized the success of the conference.

A. N. Shiryaev, I. V. Pavlov

The abstracts of the talks and presentations given at the conference are provided below.

V. I. Afanasyev (Moscow, Russia)**. Functional limit theorems for decomposable branching processes with two particle types.**

Consider a branching Galton–Watson process with particles of two types. Suppose that a particle of the first type generates descendants of both types (in the same quantities) and that a particle of the second type generates descendants of only its own type.

Let $\varphi(\,\cdot\,)$ and $\psi(\,\cdot\,)$ be generating functions of nonnegative integer random variables (r.v.'s) ξ and η. Suppose that the maximum step of distribution of the r.v. ξ is 1. We also suppose that $\mathbf{E}\xi=1$, $\mathbf{D}\xi:=\sigma_1^2\in(0,\infty)$ and $\mathbf{E}\eta=1$, $\mathbf{D}\eta:=2b_2\in(0,\infty)$.

We introduce generating functions for the progeny of particles of the first and second types, respectively, of the branching process under consideration: for $s_1,s_2\geqslant 0$,

$$f_1(s_1,s_2)=\varphi(s_1s_2),\qquad f_2(s_1,s_2)=\psi(s_2).$$

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accurate for lookback options under Lévy models in comparison to the deterministic methods from [1], [4].

The second part of this talk deals with finite difference methods for pricing American lookbacks in the Black–Scholes framework. Unlike European lookback options, American lookback options cannot be priced by closed-form formulae, even in the Black–Scholes model (see [2]), and require the use of numerical methods.

Let $U_{\mathrm{fl}}(t,x,y)$ be the price function of an American floating strike lookback put on a dividend paying stock with the maturity T conditional on $X_t = x$ and $\overline{X}_t = y$.

THEOREM 1. *Let q be a continuous dividend rate. Then the price function $U_{\mathrm{fl}}(t,x,y)$ can be represented as*

$$U_{\mathrm{fl}}(t,x,y) = e^y - e^x + e^y F(t, x-y), \qquad x \leqslant y,$$

where $F(t,x)$ is a function nondecreasing in x such that $F(T,x) = 0$, and the variational inequality

$$\begin{gathered}\max\bigl(-F, \partial_t F + 0.5\sigma^2\partial_x^2 F + (r - 0.5\sigma^2 - q)\partial_x F \\ - rF - r + qe^x\bigr) = 0, \qquad t < T, \quad x < 0, \\ 1 + F(t,0) - \frac{\partial F}{\partial x}(t,0) = 0, \qquad t < T,\end{gathered}$$

is satisfied.

We apply the Wiener–Hopf method to prove the theorem and efficiently solve the problem by an iterative finite difference scheme.

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D. F. Kuznetsov (St. Petersburg, Russia)**. Strong approximation of iterated Itô and Stratonovich stochastic integrals.**

This work continues the research started in [1] on development of efficient methods of mean-square approximation of the iterated Itô and Stratonovich stochastic integrals, which can be applied for numerical solution of Itô stochastic differential equations.

THEOREM. *Let $\psi_1(\tau), \ldots, \psi_k(\tau)$ be continuous functions on $[t,T]$, and let $\phi_j(\tau)$ be a complete orthonormal polynomial or trigonometric basis for $L_2([t,T])$, $i_1, \ldots, i_k = 0, 1, \ldots, m$. Then $I_{T,t}^k = \text{l.i.m.}_{p_1,\ldots,p_k\to\infty} I_{T,t}^{p_1\ldots p_k}$ $(k \in \mathbf{N})$, $J_{T,t}^k = \text{l.i.m.}_{p\to\infty} J_{T,t}^{k,p}$ $(k \leqslant 5)$, and, moreover,*

$$\mathbf{E}(I_{T,t}^k - I_{T,t}^{p_1\ldots p_k})^2 \leqslant k!\left(\|K\|^2 - \sum_{j_1=0}^{p_1}\cdots\sum_{j_k=0}^{p_k} C_{j_k\ldots j_1}^2\right)$$

for all $T-t\in(0,1)$, *where*

$$I_{T,t}^{k}=\int_t^T\psi_k(t_k)\cdots\int_t^{t_2}\psi_1(t_1)d\mathbf{W}_{t_1}^{(i_1)}\cdots d\mathbf{W}_{t_k}^{(i_k)},$$

$$J_{T,t}^{k}=\int_t^T\cdots\int_t^{t_2}\circ d\mathbf{W}_{t_1}^{(i_1)}\cdots\circ d\mathbf{W}_{t_k}^{(i_k)},$$

$$I_{T,t}^{p_1\ldots p_k}=\sum_{j_1=0}^{p_1}\cdots\sum_{j_k=0}^{p_k}C_{j_k\ldots j_1}\biggl(\prod_{\ell=1}^{k}\zeta_{j_\ell}^{(i_\ell)}-\underset{N\to\infty}{\text{l.i.m.}}\sum_{(\ell_1,\ldots,\ell_k)\in G_k}\prod_{s=1}^{k}\phi_{j_s}(\tau_{\ell_s})\Delta\mathbf{W}_{\tau_{\ell_s}}^{(i_s)}\biggr),$$

$$J_{T,t}^{k,p}=\sum_{j_1,\ldots,j_k=0}^{p}C_{j_k\ldots j_1}\prod_{\ell=1}^{k}\zeta_{j_\ell}^{(i_\ell)},$$

$$C_{j_k\ldots j_1}=\int_{[t,T]^k}K(t_1,\ldots,t_k)\prod_{\ell=1}^{k}\phi_{j_\ell}(t_\ell)\,dt_1\cdots dt_k;$$

$\|\cdot\|$ *is the* $L_2([t,T]^k)$*-norm*; d *and* $\circ d$ *are the Itô and Stratonovich differentials, respectively*; $K(t_1,\ldots,t_k)=I\{t_1<\cdots<t_k\}\psi_1(t_1)\ldots\psi_k(t_k)$, $\mathbf{W}_\tau^{(i)}$ $(i=1,\ldots,m)$ *are independent standard Wiener processes*; $\mathbf{W}_\tau^{(0)}=\tau$, $\Delta\mathbf{W}_{\tau_j}^{(i)}=\mathbf{W}_{\tau_{j+1}}^{(i)}-\mathbf{W}_{\tau_j}^{(i)}$, $\zeta_j^{(i)}=\int_t^T\phi_j(\tau)\,d\mathbf{W}_\tau^{(i)}$ $(i\neq0)$ *are i.i.d.* $N(0,1)$*-r.v.'s*; $t=\tau_0<\cdots<\tau_N=T$, $\max_{0\leqslant j\leqslant N-1}(\tau_{j+1}-\tau_j)\to0$ *as* $N\to\infty$; $H_k=\{(\ell_1,\ldots,\ell_k)\colon\ell_1,\ldots,\ell_k=0,1,\ldots,N-1\}$; *and* $L_k=\{(\ell_1,\ldots,\ell_k)\colon\ell_1,\ldots,\ell_k=0,1,\ldots,N-1;\ \ell_g\neq\ell_r\ (g\neq r);\ g,r=1,\ldots,k\}$, $G_k=H_k\setminus L_k$.

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K. S. Kuznetsov (St. Petersburg, Russia)**. Weighted average price management of manufacturer realization on commodity exchanges with predetermined volume of sales.**

Theorem. *We assume that* $t\in\mathbf{N}_0$ *and* Δt *correspond to the unit time interval between trading days*; *i.e.*, $t=0,1,\ldots,T$. *The observed price* $\widetilde{x}_t$ *on a commodity exchange is a realization of a stochastic process* (*see* [1], [2]), *which follows the stochastic differential equation*

$$dx_t=c_tx_t\,dt+\sigma x_t\,dW_t,$$

where c_t *is the coefficient*, W_t *is a standard Wiener process*, *and* σ *is the volatility coefficient* (*a constant*). *We also assume that the quantity of commodity units* $\widetilde{a}_t$ *sold at a certain time interval* $[0,t]$ *is given by*

$$\widetilde{a}_t=A\widetilde{x}_t+B+\widetilde{x}_tA\frac{1}{\sigma^2T}e^{\sigma^2(T-t)}-\widetilde{x}_tA\frac{1}{\sigma^2T}-\widetilde{x}_tA\frac{T-t}{T},$$

where

$$A=\frac{a_{\max}-a_{\min}}{x_{\max}-x_{\min}},\quad B=-\frac{a_{\max}-a_{\min}}{x_{\max}-x_{\min}}x_{\min}+a_{\min}$$