Dedicated to the 90th anniversary of Moscow Aviation Institute and the 100th anniversary of the birth of academician I.F. Obraztsov

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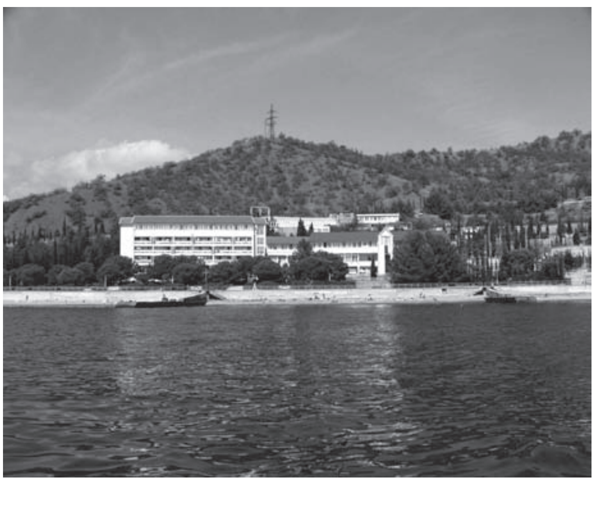

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APPLICATION OF MULTIPLE FOURIER–LEGENDRE SERIES TO THE IMPLEMENTATION OF STRONG EXPONENTIAL MILSTEIN AND WAGNER–PLATEN METHODS FOR NON-COMMUTATIVE SEMILINEAR SPDES

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1. Introduction. This work continues the research [1, 2] on methods of the mean-square approximation of Itô iterated stochastic integrals (ISIs) with respect to the infinite-dimensional Q–Wiener process. It is known that effective approach to the construction of high-order strong numerical methods (with respect to the temporal discretization) for semilinear stochastic partial differential equations (SSPDEs) is based on the Taylor formula in Banach spaces and exponential formula for the mild solution of SSPDEs [3, 4]. In [3, 4] the exponential Milstein and Wagner–Platen methods for SSPDEs with multiplicative trace class noise were constructed. Under the appropriate conditions these methods have strong orders of convergence $1{,}0-\varepsilon$ and $1{,}5-\varepsilon$ [3, 4] (here ε is an arbitrary small posilive real number) and include Itô ISIs of multiplicities 1 to 3 with respect to the infinite-dimensional Q–Wiener process. The numerical modeling of these ISIs is a difficult problem if commutativity conditions for SSPDE are not fulfilled [3, 4]. In this paper, we extend the method [5] and combine the obtained results with the results from [1, 2].

2. Approximation of ISIs From the Exponential Milstein and Wagner–Platen Schemes. Let U, H be separable $\mathbb{R}$–Hilbert spaces, let $(\Omega, \mathbf{F}, (\mathbf{F}_t)_{t\in[0,T^*]}, \mathrm{P})$ be a filtered probability space, let $\mathbf{W}_t$ be an U-valued Q–Wiener process with respect to $(\mathbf{F}_t)_{t\in[0,T^*]}$, which has a linear trace class covariance operator $Q\colon U\to U$. Let $H(U_0, H)$ be a space of Hilbert–Schmidt operators from U_0 to H, where $U_0 = Q^{1/2}U$.

Consider the parabolic SSPDE with multiplicative trace class noise

$$dX_t = (AX_t + F(X_t))dt + B(X_t)\,d\mathbf{W}_t, \quad X_0 = \xi, \quad t\in[0,T^*], \tag{1}$$

where nonlinear operators F, B ($F\colon H\to H$, $B\colon H\to H(U_0,H)$), linear operator $A\colon D(A)\subset H\to H$ as well as the initial value ξ are assumed to satisfy the conditions of existence and uniqueness of the SSPDE (1) mild solution [4] (Assumptions A1–A4).

This paper is devoted to the approximation of most complex Itô ISIs from the Milstein and Wagner–Platen schemes [3, 4] for non-commutative SSPDE (1), which have the form

$$I_{T,t}^{(1)} = \int\limits_t^T B'(Z)\Bigl(\int\limits_t^{t_2} B(Z)\,d\mathbf{W}_{t_1}\Bigr)\,d\mathbf{W}_{t_2},$$

$$I_{T,t}^{(2)} = \int\limits_t^T B'(Z)\Bigl(\int\limits_t^{t_3} B'(Z)\Bigl(\int\limits_t^{t_2} B(Z)\,d\mathbf{W}_{t_1}\Bigr)d\mathbf{W}_{t_2}\Bigr)d\mathbf{W}_{t_3},$$

$$I^{(3)}_{T,t}=\int_t^T B''(Z)\Bigl(\int_t^{t_2} B(Z)\,d\mathbf{W}_{t_1},\int_t^{t_2} B(Z)\,d\mathbf{W}_{t_1}\Bigr)d\mathbf{W}_{t_2},$$

where $0\le t<T\le T^*$, $Z\colon\Omega\to H$ is an $\mathbf{F}_t/\mathcal{B}(H)$-measurable mapping, and B',B'' are Frêchet derivatives.

Let $I^{(l)M,q}_{T,t}$ $(l=1,2,3)$ be approximations of integrals $I^{(l)}_{T,t}$ $(l=1,2,3)$ [1, 2, 6]

$$I^{(1)M,q}_{T,t}=\sum_{i_1,i_2\in J_M} B'(Z)(B(Z)e_{i_1})e_{i_2}\sqrt{\lambda_{i_1}\lambda_{i_2}}I^{(i_1i_2)q}_{(11)T,t},$$

$$I^{(2)M,q}_{T,t}=\sum_{i_1,i_2,i_3\in J_M} B'(Z)(B'(Z)(B(Z)e_{i_1})e_{i_2})e_{i_3}\sqrt{\lambda_{i_1}\lambda_{i_2}\lambda_{i_3}}I^{(i_1i_2i_3)q}_{(111)T,t},$$

$$I^{(3)M,q}_{T,t}=\sum_{i_1,i_2,i_3\in J_M} B''(Z)(B(Z)e_{i_1},B(Z)e_{i_2})e_{i_3}\sqrt{\lambda_{i_1}\lambda_{i_2}\lambda_{i_3}}\bigl(I^{(i_1i_2i_3)q}_{(111)T,t}+I^{(i_2i_1i_3)q}_{(111)T,t}+\mathbf{1}_{\{i_1=i_2\}}I^{(0i_3)q}_{(01)T,t}\bigr),$$

$$I^{(0i_1)q}_{(01)T,t}=\frac{1}{2}(T-t)^{3/2}\Bigl(\zeta^{(i_1)}_0+\frac{1}{\sqrt{3}}\zeta^{(i_1)}_1\Bigr)\quad(\text{here } q=1),$$

$$I^{(i_1i_2)q}_{(11)T,t}=\frac{1}{2}(T-t)\Bigl(\zeta^{(i_1)}_0\zeta^{(i_2)}_0+\sum_{i=1}^{q}\frac{1}{\sqrt{4i^2-1}}\bigl(\zeta^{(i_1)}_{i-1}\zeta^{(i_2)}_{i}-\zeta^{(i_1)}_{i}\zeta^{(i_2)}_{i-1}\bigr)-\mathbf{1}_{\{i_1=i_2\}}\Bigr),$$

$$I^{(i_1i_2i_3)q}_{(111)T,t}=\sum_{j_1,j_2,j_3=0}^{q} C_{j_3j_2j_1}\Bigl(\zeta^{(i_1)}_{j_1}\zeta^{(i_2)}_{j_2}\zeta^{(i_3)}_{j_3}-\mathbf{1}_{\{i_1=i_2\}}\mathbf{1}_{\{j_1=j_2\}}\zeta^{(i_3)}_{j_3}-$$
$$-\mathbf{1}_{\{i_2=i_3\}}\mathbf{1}_{\{j_2=j_3\}}\zeta^{(i_1)}_{j_1}-\mathbf{1}_{\{i_1=i_3\}}\mathbf{1}_{\{j_1=j_3\}}\zeta^{(i_2)}_{j_2}\Bigr),$$

$$C_{j_3j_2j_1}=(T-t)^{3/2}C^*_{j_3j_2j_1},\quad C^*_{j_3j_2j_1}=\frac{1}{8}\sqrt{(2j_1+1)(2j_2+1)(2j_3+1)}C^{**}_{j_3j_2j_1},$$

$$C^{**}_{j_3j_2j_1}=\int_{-1}^{1}P_{j_3}(z)\int_{-1}^{z}P_{j_2}(y)\int_{-1}^{y}P_{j_1}(x)\,dx\,dy\,dz,$$

$I^{(0i_1)q}_{(01)T,t}$, $I^{(i_1i_2)q}_{(11)T,t}$, and $I^{(i_1i_2i_3)q}_{(111)T,t}$ are approximations of Itô ISIs

$$I^{(0i_1)}_{(01)T,t}=\int_t^T\int_t^s d\tau\,d\mathbf{w}^{(i_1)}_s,\quad I^{(i_1i_2)}_{(11)T,t}=\int_t^T\int_t^s d\mathbf{w}^{(i_1)}_\tau d\mathbf{w}^{(i_2)}_s,\quad I^{(i_1i_2i_3)}_{(111)T,t}=\int_t^T I^{(i_1i_2)}_{(11)s,t}d\mathbf{w}^{(i_3)}_s,$$

where $\mathbf{1}_A$ is the indicator of the set A, $\mathbf{w}^{(i)}_s$ $(i\in J_M)$ are independent standard Brownian motions, $J_M=\bigl\{i\colon i=(l_1,\dots,l_d)$ such that $l_1,\dots,l_d\in\{1,\dots,M\}$ and $\lambda_i>0\bigr\}$ $(d\in\mathbb{N})$, λ_i and e_i $(i\in J_{+\infty})$ are eigenvalues and eigenfunctions of Q (which form an orthonormal basis of U) correspondingly, $\zeta^{(i)}_j=\int_t^T\phi_j(s)\mathbf{w}^{(i)}_s$ $(j=0,1,\dots)$ are independent standard Gaussian random variables, $\phi_j(s)$ and $P_j(s)$ $(j=0,1,\dots)$ are complete orthonormal systems of Legendre polynomials in $L_2([t,T])$ and $L_2([-1,1])$ correspondingly.

Let $H^{(2)}(U_0,H)$ and $H^{(3)}(U_0,H)$ be spaces of bilinear and 3-linear Hilbert–Schmidt operators from $U_0\times U_0$ to H and from $U_0\times U_0\times U_0$ to H correspondingly. Let $\|\cdot\|_{H^{(2)}(U_0,H)}$, and $\|\cdot\|_{H^{(3)}(U_0,H)}$ be operator norms in these spaces.

Theorem 1 [2] *Suppose that* $B'(v)(B'(v)(B(v)))$, $B''(v)(B(v),B(v))\in H^{(3)}(U_0,H)$, $B'(v)(B(v))\in H^{(2)}(U_0,H)$, *and* $B(v)\in H(U_0,H)$ *for all* $v\in H$. *Furthermore, let* $\|B(Z)Q^{-\alpha}\|_{H(U_0,H)}+\|B'(Z)(B(Z))Q^{-\alpha}\|_{H^{(2)}(U_0,H)}+\|B'(Z)(B'(Z)(B(Z)))Q^{-\alpha}\|_{H^{(3)}(U_0,H)}+$

$+ \|B''(Z)(B(Z),B(Z))Q^{-\alpha}\|_{H^{(3)}(U_0,H)} < \infty$ *with probability* 1 *for some* $\alpha > 0$ (*we suppose that Frechet derivatives* B', B'' *exist*). *Then*

$$\mathsf{M}\|I_{T,t}^{(1)} - I_{T,t}^{(1)M,q}\|_H^2 \le (T-t)^2\left(C_1(\mathrm{Tr}Q)^2\left(\frac{1}{2} - \sum_{j=1}^{q}\frac{1}{4j^2-1}\right) + K_Q\left(\sup_{i\in J_{+\infty}\setminus J_M}\lambda_i\right)^{2\alpha}\right),$$

$$\mathsf{M}\|I_{T,t}^{(2)} - I_{T,t}^{(2)M,q}\|_H^2 + \mathsf{M}\|I_{T,t}^{(3)} - I_{T,t}^{(3)M,q}\|_H^2 \le$$

$$\le (T-t)^3\left(C_2(\mathrm{Tr}Q)^3\left(\frac{1}{6} - \sum_{j_1,j_2,j_3=0}^{q}(C^*_{j_3j_2j_1})^2\right) + L_Q\left(\sup_{i\in J_{+\infty}\setminus J_M}\lambda_i\right)^{2\alpha}\right),$$

where $q \in \mathbb{N}$, $C_1, C_2, K_Q, L_Q < \infty$, *and* $\mathrm{Tr}Q = \sum_{i\in J_{+\infty}}\lambda_i < \infty$.

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TOWARDS NUMERICAL SOLUTION OF ONE NON-CONVEX MC CONTROL PROBLEM

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Systems described by Markov chains (MC) are known long ago and are used in various fields. The control problems for Markov chains in continuous time usually can be reduced to the solution of a system of ordinary differential equations, which is an analog of the dynamic programming equation. This system contains, as an obligatory element, the solution of the minimization problem on the set of admissible controls at each instant of time and the success in obtaining a numerical solution depends on how effective this optimization procedure is, because it must be performed for all MC states at any time. The problem statement for control of connected MC and derivation of dynamic programming equation in tensor form had been presented in our previous works (see, for example [1, 2]).