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Application of Multiple FourierLegendre Series to the Implementation of Strong Exponential Milstein and WagnerPlaten Methods for Non-Commutative Semilinear SPDEs with Nonlinear Multiplicative Trace Class Noise

Dmitriy F. Kuznetsov[a,*]

[a] *Peter the Great Saint-Petersburg Polytechnic University, Saint-Petersburg, Russian Federation*

Abstract

This work is devoted to the mean-square approximation of iterated Itô stochastic integrals with respect to the infinite-dimensional Q-Wiener process. These integrals are part of the high-order strong numerical methods (with respect to the temporal discretization) for semilinear stochastic partial differential equations with nonlinear multiplicative trace class noise, which are based on the Taylor formula in Banach spaces and exponential formula for the mild solution of semilinear stochastic partial differential equations. For the approximation of the mentioned stochastic integrals we use the multiple FourierLegendre series converging in the sense of norm in Hilbert space.

Keywords: Semilinear stochastic partial differential equation, Infinite-dimensional Q-Wiener process, Nonlinear multiplicative trace class noise, Iterated Itô stochastic integral, Generalized multiple Fourier series, Multiple FourierLegendre series, Exponential Milstein scheme, Exponential WagnerPlaten scheme, Legendre polynomial, Mean-square approximation, Expansion

1. Introduction

This work continues the research [1], [2] on methods of the mean-square approximation of iterated stochastic integrals (ISIs) with respect to the infinite-dimensional Q-Wiener process. It is well known that effective approach to the construction of high-order strong numerical methods (with respect to the temporal discretization) for semilinear stochastic partial differential equations (SSPDEs) is based on the Taylor formula in Banach spaces and exponential formula for the mild solution of SSPDEs [3], [4]. In [3], [4] the exponential Milstein and Wagner–Platen methods for SSPDEs with nonlinear multiplicative trace class noise were constructed. Under the appropriate conditions these methods have strong convergence orders $1.0-\varepsilon$ and $1.5-\varepsilon$ [3], [4] (here ε is an arbitrary small posilive real number) and include Itô ISIs of multiplicities 1 to 3 with respect to the infinite-dimensional Q-Wiener process. The numerical modeling of these ISIs is a difficult problem if commutativity conditions for SSPDE are not fulfilled [3], [4]. In this paper, we extend the method [5] and combine the obtained results with the results from [1], [2].

*Corresponding author

Email address: `sde_kuznetsov.inbox.ru` (Dmitriy F. Kuznetsov)

2. Approximation of ISIs From the Exponential Milstein and Wagner–Platen Schemes

Let U, H be separable $\mathbf{R}$-Hilbert spaces, let $(\Omega, \mathbf{F}, \{\mathbf{F}_t\}_{t\in[0,S]}, \mathsf{P})$ be a filtered probability space, let $\mathbf{W}_t$ be an U-valued Q-Wiener process with respect to $\{\mathbf{F}_t\}_{t\in[0,S]}$, which has a linear trace class covariance operator $Q : U \to U$. Let $L_{\mathrm{HS}}(U_0, H)$ be a space of Hilbert-Schmidt operators mapping from U_0 to H, where $\mathbf{R}$-Hilbert space U_0 is defined as follows $U_0 = Q^{1/2}U$.

Consider the parabolic SSPDE with nonlinear multiplicative trace class noise

$$dX_t = (AX_t + F(X_t))\, dt + B(X_t)d\mathbf{W}_t, \quad X_0 = \xi, \quad t \in [0, S], \tag{1}$$

where nonlinear operators F, B ($F : H \to H$, $B : H \to L_{\mathrm{HS}}(U_0, H)$), linear operator $A : D(A) \subset H \to H$ as well as the initial value ξ are assumed to satisfy the conditions of existence and uniqueness of the mild solution of the SSPDE (1) [4] (Assumptions A1–A4).

This paper is devoted to the approximation of the most complex Itô ISIs from the Milstein and Wagner–Platen schemes [3], [4] for the non-commutative SSPDE (1), which have the form

$$I^{(1)}_{T,t} = \int_t^T B'(Z)\left(\int_t^{t_2} B(Z)d\mathbf{W}_{t_1}\right)d\mathbf{W}_{t_2},$$

$$I^{(2)}_{T,t} = \int_t^T B'(Z)\left(\int_t^{t_3} B'(Z)\left(\int_t^{t_2} B(Z)d\mathbf{W}_{t_1}\right)d\mathbf{W}_{t_2}\right)d\mathbf{W}_{t_3},$$

$$I^{(3)}_{T,t} = \int_t^T B''(Z)\left(\int_t^{t_2} B(Z)d\mathbf{W}_{t_1}, \int_t^{t_2} B(Z)d\mathbf{W}_{t_1}\right)d\mathbf{W}_{t_2},$$

where $0 \le t < T \le S$, $Z : \Omega \to H$ is an $\mathbf{F}_t/\mathcal{B}(H)$-measurable mapping, and B', B'' are Frêchet derivatives.

Let $I^{(l)M,q_l}_{T,t}$ $(l = 1, 2, 3)$ be approximations of the integrals $I^{(l)}_{T,t}$ $(l = 1, 2, 3)$ [1], [2], [6]

$$I^{(1)M,q_1}_{T,t} = \sum_{i_1,i_2\in J_M} B'(Z)\left(B(Z)e_{i_1}\right)e_{i_2}\sqrt{\lambda_{i_1}\lambda_{i_2}}\, I^{(i_1i_2)q_1}_{(11)T,t},$$

$$I^{(2)M,q_2}_{T,t} = \sum_{i_1,i_2,i_3\in J_M} B'(Z)\left(B'(Z)\left(B(Z)e_{i_1}\right)e_{i_2}\right)e_{i_3}\sqrt{\lambda_{i_1}\lambda_{i_2}\lambda_{i_3}}\, I^{(i_1i_2i_3)q_2}_{(111)T,t},$$

$$I^{(3)M,q_3}_{T,t} = \sum_{i_1,i_2,i_3\in J_M} B''(Z)\left(B(Z)e_{i_1}, B(Z)e_{i_2}\right)e_{i_3}\sqrt{\lambda_{i_1}\lambda_{i_2}\lambda_{i_3}}\left(I^{(i_1i_2i_3)q_3}_{(111)T,t} + I^{(i_2i_1i_3)q_3}_{(111)T,t} + \mathbf{1}_{\{i_1=i_2\}} I^{(0i_3)1}_{(01)T,t}\right),$$

where

$$I^{(0i_3)1}_{(01)T,t} = \frac{1}{2}(T-t)^{3/2}\left(\zeta_0^{(i_3)} + \frac{1}{\sqrt{3}}\zeta_1^{(i_3)}\right),$$

$$I^{(i_1i_2)q_1}_{(11)T,t} = \frac{1}{2}(T-t)\left(\zeta_0^{(i_1)}\zeta_0^{(i_2)} + \sum_{i=1}^{q_1}\frac{1}{\sqrt{4i^2-1}}\left(\zeta_{i-1}^{(i_1)}\zeta_i^{(i_2)} - \zeta_i^{(i_1)}\zeta_{i-1}^{(i_2)}\right) - \mathbf{1}_{\{i_1=i_2\}}\right),$$

$$I^{(i_1i_2i_3)q_2}_{(111)T,t} = \sum_{j_1,j_2,j_3=0}^{q_2} C_{j_3j_2j_1}\Biggl(\zeta_{j_1}^{(i_1)}\zeta_{j_2}^{(i_2)}\zeta_{j_3}^{(i_3)} - \mathbf{1}_{\{i_1=i_2\}}\mathbf{1}_{\{j_1=j_2\}}\zeta_{j_3}^{(i_3)} -$$

$$-\mathbf{1}_{\{i_2=i_3\}}\mathbf{1}_{\{j_2=j_3\}}\zeta_{j_1}^{(i_1)} - \mathbf{1}_{\{i_1=i_3\}}\mathbf{1}_{\{j_1=j_3\}}\zeta_{j_2}^{(i_2)}\Biggr), \tag{2}$$

$$C_{j_3 j_2 j_1} = (T-t)^{3/2} G_{j_3 j_2 j_1},\ \ G_{j_3 j_2 j_1} = \frac{1}{8}\sqrt{(2j_1+1)(2j_2+1)(2j_3+1)}L_{j_3 j_2 j_1},$$

$$L_{j_3 j_2 j_1} = \int_{-1}^{1} P_{j_3}(z)\int_{-1}^{z} P_{j_2}(y)\int_{-1}^{y} P_{j_1}(x)dxdydz,$$

$I_{(01)T,t}^{(0i_3)1}$, $I_{(11)T,t}^{(i_1 i_2)q_1}$, and $I_{(111)T,t}^{(i_1 i_2 i_3)q_2}$ are approximations of Itô ISIs

$$I_{(01)T,t}^{(0i_3)} = \int_t^T\int_t^s d\tau d\mathbf{w}_s^{(i_3)},\ I_{(11)T,t}^{(i_1 i_2)} = \int_t^T\int_t^s d\mathbf{w}_\tau^{(i_1)}d\mathbf{w}_s^{(i_2)},\ I_{(111)T,t}^{(i_1 i_2 i_3)} = \int_t^T I_{(11)s,t}^{(i_1 i_2)}d\mathbf{w}_s^{(i_3)},$$

$\mathbf{1}_A$ is the indicator of the set A, $\mathbf{w}_s^{(i)}$ $(i \in J_M)$ are independent standard Wiener processes, $J_M = \{i : i = (l_1,\ldots,l_d)$ and $l_1,\ldots,l_d \in \{1,\ldots,M\},\ \lambda_i > 0\}$, $J_{+\infty} \stackrel{\mathsf{def}}{=} J$, $d \in \mathbf{N}$, λ_i and e_i $(i \in J)$ are eigenvalues and eigenfunctions of Q (which form an orthonormal basis of U) correspondingly, $\zeta_j^{(i)} = \int_t^T \phi_j(s)\mathbf{w}_s^{(i)}$ $(j = 0,1,\ldots)$ are i.i.d. $N(0,1)$-r.v.'s, $\phi_j(s)$ and $P_j(s)$ $(j = 0,1,\ldots)$ are complete orthonormal systems of Legendre polynomials in $L_2([t,T])$ and $L_2([-1,1])$ correspondingly.

Let $L_{\mathrm{HS}}^{(k)}(U_0,H)$ $(k \ge 2)$ be the space of k-linear Hilbert–Schmidt operators from $U_0 \times \ldots \times U_0$ (k times) to H. Let $\|\cdot\|_{L_{\mathrm{HS}}^{(k)}(U_0,H)}$ be operator norm in this space.

Theorem 1. [2], [6] *Suppose that* $B(v) \in L_{\mathrm{HS}}(U_0,H)$ *and*

$$B'(v)(B(v)) \in L_{\mathrm{HS}}^{(2)}(U_0,H), \quad B'(v)(B'(v)(B(v))),\ B''(v)(B(v),B(v)) \in L_{\mathrm{HS}}^{(3)}(U_0,H)$$

for all $v \in H$. *Furthermore, let*

$$\left\|B(Z)Q^{-\alpha}\right\|_{L_{\mathrm{HS}}(U_0,H)} + \left\|B'(Z)(B(Z))Q^{-\alpha}\right\|_{L_{\mathrm{HS}}^{(2)}(U_0,H)} + \left\|B'(Z)(B'(Z)(B(Z)))Q^{-\alpha}\right\|_{L_{\mathrm{HS}}^{(3)}(U_0,H)} +$$

$$+\left\|B''(Z)(B(Z),B(Z))Q^{-\alpha}\right\|_{L_{\mathrm{HS}}^{(3)}(U_0,H)} < \infty$$

with probability 1 *for some* $\alpha > 0$ (*we suppose that Frêchet derivatives* B', B'' *exist*). *Then*

$$\mathsf{M}\left\|I_{T,t}^{(1)} - I_{T,t}^{(1)M,p}\right\|_H^2 \le (T-t)^2\Biggl(C_1\,(\mathrm{Tr}Q)^2\left(\frac{1}{2} - \sum_{j=1}^{p}\frac{1}{4j^2-1}\right) + K_Q\Bigl(\sup_{i\in J\backslash J_M}\lambda_i\Bigr)^{2\alpha}\Biggr),$$

$$\mathsf{M}\left\|I_{T,t}^{(2)} - I_{T,t}^{(2)M,q}\right\|_H^2 + \mathsf{M}\left\|I_{T,t}^{(3)} - I_{T,t}^{(3)M,q}\right\|_H^2 \le$$

$$\le (T-t)^3\Biggl(C_2\,(\mathrm{Tr}Q)^3\left(\frac{1}{6} - \sum_{j_1,j_2,j_3=0}^{q}(G_{j_3 j_2 j_1})^2\right) + L_Q\Bigl(\sup_{i\in J\backslash J_M}\lambda_i\Bigr)^{2\alpha}\Biggr),$$

where $q \in \mathbf{N}$, $M, C_1, C_2, K_Q, L_Q < \infty$, $\mathrm{Tr}Q = \sum\limits_{i\in J}\lambda_i < \infty$.

Note that $q \ll p$ if $T - t \ll 1$. The proof of Theorem 1 is based on Theorems 2, 3 (see below). Consider the following Itô ISIs

$$J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)} = \int_t^T \psi_k(t_k)\ldots\int_t^{t_2}\psi_1(t_1)d\mathbf{w}_{t_1}^{(i_1)}\ldots d\mathbf{w}_{t_k}^{(i_k)},$$

where $0 \le t < T \le S$, $\psi_l(\tau)$ $(l = 1, \ldots, k)$ are nonrandom functions on $[t, T]$, $\mathbf{w}_\tau^{(i)}$ $(i = 1, \ldots, m)$ are independent standard Wiener processes, $\mathbf{w}_\tau^{(0)} = \tau$, $i_1, \ldots, i_k = 0, 1, \ldots, m$.

Let

$$\sum_{\substack{(\{\{g_1,g_2\},\ldots,\{g_{2r-1},g_{2r}\}\},\{q_1,\ldots,q_{k-2r}\})\\ \{g_1,g_2,\ldots,g_{2r-1},g_{2r},q_1,\ldots,q_{k-2r}\}=\{1,2,\ldots,k\}}}$$

be the sum with respect to all possible permutations of the set

$$(\{\{g_1, g_2\}, \ldots, \{g_{2r-1}, g_{2r}\}\}, \{q_1, \ldots, q_{k-2r}\}),$$

where $\{g_1, g_2, \ldots, g_{2r-1}, g_{2r}, q_1, \ldots, q_{k-2r}\} = \{1, 2, \ldots, k\}$, braces mean an unordered set, and parentheses mean an ordered set.

Theorem 2. [6] (*also see* [1], [2]). *Suppose that $\psi_l(\tau)$ $(l = 1, \ldots, k)$ are continuous nonrandom functions on $[t, T]$ and $\{\phi_j(x)\}_{j=0}^{\infty}$ is a complete orthonormal system of continuous functions in $L_2([t, T])$. Then $J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)} = \underset{p_1,\ldots,p_k\to\infty}{\text{l.i.m.}} J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)p_1,\ldots,p_k}$ and*

$$\mathsf{M}\left(J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)} - J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)p_1,\ldots,p_k}\right)^2 \le k!\left(I_k - \sum_{j_1=0}^{p_1}\ldots\sum_{j_k=0}^{p_k} C_{j_k\ldots j_1}^2\right), \tag{3}$$

where

$$J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)p_1,\ldots,p_k} = \sum_{j_1=0}^{p_1}\ldots\sum_{j_k=0}^{p_k} C_{j_k\ldots j_1}\left(\prod_{l=1}^{k}\zeta_{j_l}^{(i_l)} + \sum_{r=1}^{[k/2]}(-1)^r\times\right.$$

$$\left.\times\sum_{\substack{(\{\{g_1,g_2\},\ldots,\{g_{2r-1},g_{2r}\}\},\{q_1,\ldots,q_{k-2r}\})\\ \{g_1,g_2,\ldots,g_{2r-1},g_{2r},q_1,\ldots,q_{k-2r}\}=\{1,2,\ldots,k\}}}\prod_{s=1}^{r}\mathbf{1}_{\{i_{g_{2s-1}}=\,i_{g_{2s}}\neq 0\}}\mathbf{1}_{\{j_{g_{2s-1}}=\,j_{g_{2s}}\}}\prod_{l=1}^{k-2r}\zeta_{j_{q_l}}^{(i_{q_l})}\right), \tag{4}$$

l.i.m. *is a limit in the mean-square sense, $i_1, \ldots, i_k = 0, 1, \ldots, m$, $\zeta_j^{(i)} = \int_t^T \phi_j(s)d\mathbf{w}_s^{(i)}$ are i.i.d. $N(0,1)$-r.v.'s for various i or j (if $i \neq 0$), in* (3): *$T - t \in (0, +\infty)$ for $i_1, \ldots, i_k = 1, \ldots, m$ and $T - t \in (0, 1)$ for $i_1, \ldots, i_k = 0, 1, \ldots, m$, $C_{j_k\ldots j_1}$ is the Fourier coefficient*

$$C_{j_k\ldots j_1} = \int_{[t,T]^k} K(t_1, \ldots, t_k)\prod_{l=1}^{k}\phi_{j_l}(t_l)dt_1\ldots dt_k,$$

$$I_k = \int_{[t,T]^k} K^2(t_1, \ldots, t_k)dt_1\ldots dt_k,$$

$K(t_1, \ldots, t_k) = \psi_1(t_1)\ldots\psi_k(t_k)\mathbf{1}_{\{t_1<\ldots<t_k\}}$ for $k \ge 2$ and $K(t_1) \equiv \psi_1(t_1)$, here and in (4): *$\mathbf{1}_A$ is the indicator of the set A, $[x]$ is an integer part of $x \in \mathbf{R}$.*

Note that from (4) if $k = 3$, $\psi_1(\tau), \ldots, \psi_3(\tau) \equiv 1$, $p_1 = \ldots = p_3 = q_2$ we obtain (2).

Consider the Itô ISIs with respect to the Q-Wiener process in the form

$$I[\Phi^{(k)}, \psi^{(k)}]_{T,t} = \int_t^T \Phi_k(Z)\left(\ldots\left(\int_t^{t_2}\Phi_1(Z)\psi_1(t_1)d\mathbf{W}_{t_1}\right)\ldots\right)\psi_k(t_k)d\mathbf{W}_{t_k}, \tag{5}$$

where $Z : \Omega \to H$ is an $\mathbf{F}_t/\mathcal{B}(H)$-measurable mapping, $\psi_l(\tau)$ $(l = 1, \ldots, k)$ are the same functions as in Theorem 2, and $\Phi_k(v)(\ldots(\Phi_1(v))\ldots) \in L_{\mathrm{HS}}^{(k)}(U_0, H)$ for all $v \in H$.

Let $I[\Phi^{(k)}, \psi^{(k)}]_{T,t}^{M}$ be an approximation of ISI (5)

$$I[\Phi^{(k)}, \psi^{(k)}]_{T,t}^{M} = \sum_{i_1,\ldots,i_k \in J_M} \Phi_k(Z)\left(\ldots\left(\Phi_1(Z)e_{i_1}\right)\ldots\right) e_{i_k} \prod_{l=1}^{k} \sqrt{\lambda_{i_l}}\, J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)}, \tag{6}$$

and let $I[\Phi^{(k)}, \psi^{(k)}]_{T,t}^{M,p_1\ldots,p_k}$ be an approximation of ISI (6)

$$I[\Phi^{(k)}, \psi^{(k)}]_{T,t}^{M,p_1\ldots p_k} = \sum_{i_1,\ldots,i_k \in J_M} \Phi_k(Z)\left(\ldots\left(\Phi_1(Z)e_{i_1}\right)\ldots\right) e_{i_k} \prod_{l=1}^{k} \sqrt{\lambda_{i_l}}\, J[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)p_1,\ldots,p_k}, \tag{7}$$

where $0 \le t < T \le S$, $i_1, \ldots, i_k \in J_M$, $M < \infty$.

Theorem 3. [1], [2], [6]. *Let the conditions of Theorem 2 be fulfilled. Moreover, let $Q : U \to U$ is a linear, nonnegative and symmetric trace class operator (λ_i and e_i $(i \in J)$ are its eigenvalues and eigenfunctions correspondingly), $\mathbf{W}_\tau$, $\tau \in [0, S]$ is an U-valued Q-Wiener process, $Z : \Omega \to H$ is an $\mathbf{F}_t/\mathcal{B}(H)$-measurable mapping, $\Phi_k(v)(\ldots(\Phi_1(v))\ldots) \in L_{\mathrm{HS}}^{(k)}(U_0, H)$ for all $v \in H$, i.e. $\|\Phi_k(Z)\left(\ldots\left(\Phi_1(Z)e_{i_1}\right)\ldots\right) e_{i_k}\|_H^2 \le L_k < \infty$ with probability 1 for all $i_1, \ldots, i_k \in J_M$, $M \in \mathbf{N}$. Then*

$$\mathsf{M}\left\|I[\Phi^{(k)}, \psi^{(k)}]_{T,t}^{M} - I[\Phi^{(k)}, \psi^{(k)}]_{T,t}^{M,p_1\ldots p_k}\right\|_H^2 \le L_k (k!)^2 \left(\mathrm{Tr}\ Q\right)^k \left(I_k - \sum_{j_1=0}^{p_1} \ldots \sum_{j_k=0}^{p_k} C_{j_k\ldots j_1}^2\right), \tag{8}$$

where $M < \infty$, $\mathrm{Tr}Q = \sum\limits_{i \in J} \lambda_i < \infty$.

It should be noted that the right-hand side of the inequality (8) is independent of M and tends to zero if $p_1, \ldots, p_k \to \infty$ due to the Parseval's equality.

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Российский университет дружбы народов
115419, ГСП-1, г. Москва, ул. Орджоникидзе, д. 3

Типография РУДН
115419, ГСП-1, г. Москва, ул. Орджоникидзе, д. 3, тел. 952-04-41