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ABSTRACTS OF TALKS GIVEN AT THE 10TH INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS*

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The Tenth International Conference on Stochastic Methods (ICSM-10) was held May 31–June 6, 2025 in Divnomorskoe (near the town of Gelendzhik) at the Raduga sports and fitness center of Don State Technical University, where the previous conference (ICSM-9) took place. ICSM-10 was organized by the Steklov Mathematical Institute of Russian Academy of Sciences (Steklov International Mathematical Center); Lomonosov Moscow State University (Department of Probability Theory); Artificial Intelligence Research Institute, National Committee of the Bernoulli Society of Mathematical Statistics, Probability Theory, Combinatorics, and Applications; Regional Mathematical Center of Southern Federal University; and Don State Technical University (Department of Higher Mathematics).

This conference was dedicated to the 90th anniversary of its founding by A. N. Kolmogorov of the Department of Probability Theory at Lomonosov Moscow State University. A. N. Shiryaev, Full Member of the Russian Academy of Sciences and Chairman of the Organizing Committee of the ICSM-10, in his introductory lecture "The Ninetieth Anniversary of the Founding of the Department of Probability Theory at Lomonosov Moscow State University," spoke in detail about the development of probability theory in the USSR as a separate science and how the foundation was laid by 1935 for the creation of the Department of Probability Theory in the Mathematical Department of the Faculty of Mechanics and Mathematics of Lomonosov Moscow State University. In this lecture he also presented and analyzed the outstanding scientific achievements of the department's staff throughout the 90 years of its existence.[1]

A special issue of *Journal of Mathematical Sciences* dedicated to the 90th anniversary of Academician A. N. Shiryaev was prepared and published before the opening of the conference. A. N. Shiryaev has served as the head of the Department of Probability Theory at Moscow State University for many years. Professor A. N. Karapetyants, the head of the Regional Mathematical Center of Southern Federal University, delivered a welcoming speech at the opening of the conference and distributed several copies of the special issue of the journal to the conference organizing committee.

Many leading scientists from Russia, Portugal, Tajikistan, Kazakhstan, and Uzbekistan took part in ICSM-10. Russian participants came from Moscow, St. Petersburg, Rostov-on-Don, Voronezh, Tomsk, Ufa, Nizhny Novgorod, Khabarovsk, Chelyabinsk, Sochi, Taganrog, Maikop, Veliky Novgorod, Ul'yanovsk, Syktyvkar, Krasnodar, Archangel'sk, Ivanovo, and Kaluga. Twenty-two talks were given at joint sessions, and fifty-four talks were presented at parallel sessions.

I. V. Pavlov, T. A. Volosatova, P. A. Yaskov,
A. A. Muravlev, A. N. Karapetyants

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[1]A transcript of this lecture appears in the present issue of *Theory of Probability and Its Applications*; see pp. 498–501.

B. M. Kushnarenko, R. A. Firsov (Lomonosov Moscow State University, VEGA Institute, Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow, Russia). **On some extensions of Lorden's inequality.**

The classical Lorden's inequality (1970) gives an upper bound for $\sup_{t\geqslant 0}\mathbf{E}R_t$, where $R_t = S_{N(t)} - t$ is the overshoot of a one-dimensional random walk over the level t.

Let $(Z_k)_{k\geqslant 1}$ be i.i.d. vectors from $\mathbf{R}^d$ with nonnegative coordinates. For a fixed norm $\|\cdot\|$, we set

$$S_0 = 0, \quad S_n = \sum_{k=1}^{n} Z_k;$$

$$N(t) = \inf\{n \geqslant 1 \colon \|S_n\| > t\}, \qquad R_t = \|S_{N(t)}\| - t.$$

Assume that

$$\mathbf{E}\|Z_1\|_1 > 0, \qquad \mathbf{E}\|Z_1\|_1^2 < \infty, \qquad \mathbf{E}\|Z_1\|^2 < \infty.$$

Theorem 1. *Let there exist $C > 0$ such that $C\|x\|_1 \leqslant \|x\|$ for each $x \in \mathbf{R}^d$. Then*

$$\sup_{t\geqslant 0}\mathbf{E}R_t \leqslant \left(\frac{1}{C} + \frac{1}{\|\mathbf{1}\|}\right)\frac{\mathbf{E}\|Z_1\|^2 + \mathbf{E}\|Z_1\|_1^2}{\mathbf{E}\|Z_1\|_1}.$$

Theorem 2. *Let $\|\cdot\| = \|\cdot\|_2$ and $d \geqslant 1$. Then*

$$\sup_{t\geqslant 0}\mathbf{E}R_t \leqslant 2\left(\sqrt{d} + \frac{1}{\sqrt{d}}\right)\frac{\mathbf{E}\|Z_1\|_1^2}{\mathbf{E}\|Z_1\|_1}.$$

The estimate in Theorem 2 is asymptotically optimal in its order with respect to the dimension d and, for $d = 1$, reduces to Lorden's classical result. Since the function $t \mapsto \mathbb{E}R_t$ is subadditive, and by the law of large numbers, we have the asymptotic formula

$$\frac{\mathbf{E}R_t}{t} \leqslant \left(\frac{\mathbf{E}\|Z_1\|}{\|\mathbf{E}Z_1\|} - 1\right) + o(1), \qquad t \to \infty.$$

The above inequalities are useful, for instance, in portfolio-risk assessment. A passage from a scalar risk measure to a vector of returns $(Z_k^{(1)}, \dots, Z_k^{(d)})$ suggests naturally the use of the ℓ^1-norm. In particular, Theorem 2 provides an upper guarantee for the portfolio draw-down under a price shock of size t.

D. F. Kuznetsov (Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia). **Latest results on a new approach to series expansion of iterated Stratonovich stochastic integrals. Multiplicities from 1 to 8 and beyond.**

Theorem 1 (see [1, sections 2.22, 2.27, 2.29–2.31]). *Let $\psi_1(\tau), \dots, \psi_k(\tau) \in C[t, T]$, $\{\phi_j(x)\}_{j=0}^{\infty}$ be an arbitrary complete orthonormal system in $L_2[t, T]$, and let one of the conditions* (2.1294), (2.1310), (2.1341) *from* [1] *be met. Then, for all $i_1, \dots, i_k = 0, 1, \dots, m$ and $k \in \mathbf{N}$,*

$$\int_t^T \psi_k(t_k) \cdots \int_t^{t_2} \psi_1(t_1) \circ d\mathbf{W}_{t_1}^{(i_1)} \cdots \circ d\mathbf{W}_{t_k}^{(i_k)}$$

$$= \underset{p\to\infty}{\text{l.i.m.}} \sum_{j_1,\dots,j_k=0}^{p} C_{j_k\dots j_1} \prod_{l=1}^{k} \zeta_{j_l}^{(i_l)}, \tag{1}$$

where $\zeta_j^{(i)} = \int_t^T \phi_j(\tau)\, d\mathbf{W}_\tau^{(i)}$ *are independent* $N(0,1)$*-r.v.'s* $(i \neq 0)$, $C_{j_k \ldots j_1}$ *is the Fourier coefficient for*

$$K(t_1,\ldots,t_k) = \psi_1(t_1)\cdots\psi_k(t_k)\,\mathbf{I}_{\{t_1<\cdots<t_k\}} \quad (k \geqslant 2),$$
$$K(t_1) = \psi_1(t_1), \qquad t_1,\ldots,t_k \in [t,T],$$

$d\mathbf{W}_\tau^{(i)}$ *and* $\circ d\mathbf{W}_\tau^{(i)}$ *are the Itô and Stratonovich differentials, and* $\mathbf{W}_\tau^{(i)}$, $i=1,\ldots,m$, *are independent standard Wiener processes,* $\mathbf{W}_\tau^{(0)} = \tau$.

Theorem 2 (see [1, sections 2.1.4, 2.24, 2.32–2.34]). *Let* $\{\phi_j(x)\}_{j=0}^\infty$ *be as in Theorem* 1. *Then* (1) *holds without assumptions* (2.1294), (2.1310), (2.1341) *from* [1] *in the following three cases*:

(1) $\psi_1(\tau), \psi_2(\tau) \in C[t,T]$ $(k=1,2)$;
(2) $\psi_1(\tau) = (\tau-t)^{p_1},\ldots,\psi_4(\tau) = (\tau-t)^{p_4}$, $p_1,\ldots,p_4 = 0,1,2,\ldots$ $(k=3,4)$;
(3) $\psi_1(\tau),\ldots,\psi_6(\tau) \equiv 1$ $(k=5,6)$.

Theorem 3 (see [1, sections 2.36, 2.37]). *Let* $\{\phi_j(x)\}_{j=0}^\infty$ *be a complete orthonormal system of Legendre polynomials or the trigonometric Fourier basis in* $L_2[t,T]$. *Then* (1) *holds without assumptions* (2.1294), (2.1310), (2.1341) *from* [1] *for the case* $\psi_1(\tau),\ldots,\psi_8(\tau) \equiv 1$ $(k=7,8)$.

The above theorems can be used for numerical solution of stochastic ODEs with noncommutative noise.

REFERENCES

[1] D. F. Kuznetsov, *Strong Approximation of Iterated Itô and Stratonovich Stochastic Integrals Based on Generalized Multiple Fourier Series. Application to Numerical Solution of Itô SDEs and Semilinear SPDEs*, preprint, https://arxiv.org/abs/2003.14184v53, 2025.

A. V. Lebedev (Lomonosov Moscow State University, Moscow, Russia)**. Three-dimensional polynomial copulas and nontransitive tuples of random variables.**

This talk continues the studies begun in [1]. Consider a random vector (X_1,X_2,X_3) with polynomial density on $[0,1]^3$ of the form

$$p(x_1,x_2,x_3) = 1 + \sum_{i=1}^3 a_i x_i + \sum_{j=1}^3 b_i x_i^2 + \sum_{i=1}^3\sum_{j=1}^3 c_{ij} x_i x_j^2, \tag{1}$$

where C is a 3×3-matrix with diagonal entries $c_{ii}=0$, $1 \leqslant i \leqslant 3$, $p(x_1,x_2,x_3) \geqslant 0$.

Theorem. *Density* (1) *with skew-symmetric matrix* C *defines a copula under the conditions*

$$a_i = -\frac{c_{ij}+c_{ik}}{3}, \qquad b_i = -\frac{c_{ji}+c_{ki}}{2},$$

where i, j, k *are distinct*, $1 \leqslant i,j,k \leqslant 3$; *it is nontransitive if and only if* $C = AC^0$, *where* A *is a positive number, and* C^0 *is the skew-symmetric matrix with elements*

$$c_{12}^0 = \frac{10}{32}w_1 + \frac{11}{32}w_2 + \frac{11}{32}w_3,$$
$$c_{23}^0 = \frac{11}{32}w_1 + \frac{10}{32}w_2 + \frac{11}{32}w_3,$$
$$c_{31}^0 = \frac{11}{32}w_1 + \frac{11}{32}w_2 + \frac{10}{32}w_3$$